

Introduction

The book is intended to serve as a manual for a half-year introductory course of number theory. Here “introductory” means the first lecture course in number theory the students will take. However, we do not assume that any more advanced courses will follow—on the contrary, we suppose that, for most listeners, this will be the only contact with number theory during their university studies. Therefore, our aim is to present the theory to the extent which in our view is indispensable to every mathematics student, regardless of their future specialization. However, we hope that some readers will not restrict themselves to this introductory part, but continue their study of that discipline due to its beauty, depth and intriguing perspectives.

In this book, we have set some target points, important for number theory due to the methods used and essential to the development of the theory. These include the local methods, the Quadratic Reciprocity Law, the Prime Number Theorem, unique factorization of ideals in rings of algebraic integers in finite extensions of \mathbb{Q} , units in the rings of integers in quadratic extensions of \mathbb{Q} , and decomposition of primes in rings of integers. This choice of targets enables presenting the most important problems of number theory and the range of methods used, algebraic, analytic and geometric.

It is clear to the authors that although the very restricted framework may leave the reader unsatisfied, extending the material e.g. by presenting the details of the proofs that are only sketched cannot be realized within a one-semester course. To some extent, producing this feeling of dissatisfaction was intentional: we wanted to show that the results presented are not complete and give vast opportunities for further study and research.

This book is based on the lecture notes (in English) that the first author distributed among the listeners of his 2011–2012 lectures at Warsaw University. The second author corrected and complemented these notes in various ways, and also added the problem sections.

According to our experience, with some self-restraint of the lecturer together with relegating some parts of the material to classes, this material can be realized within a one-semester course (two hours of lectures and two

hours of classes a week). We hope that our book will be useful for the listeners of introductory lectures devoted to number theory, sparing them the time of finding the necessary facts in the vast and sometimes not very accessible or easily readable literature.

In principle, the material of this book does not go beyond the 19th century mathematics. In no case, do we claim any originality of the results presented. For the most part, the results are well-known, and so giving the exact sources does not seem necessary; we indicate them only exceptionally. Several fragments of the book were influenced by well-known number theory monographs, including [4], [7], [10], [11]; this is often indicated in the text at appropriate places.

The only original element of this book is the choice and arrangement of the material; the authors hope that it will help realize the targets of introductory number theory lectures, as set out at the beginning of this introduction.

The beauty of number theory lies in the fact that on the one hand it enriches other domains of mathematics, and on the other hand it makes frequent use of their methods and results. It was our intention to emphasize these two aspects and not to limit the problems or methods presented to those traditionally attributed to number theory. It is our view that all facts and methods elaborated in other areas of mathematics are allowed in a number theory course. This causes certain problems in determining the necessary prerequisites the reader should know. These certainly include some algebra background (elements of the theory of rings and modules) and elements of calculus (real and complex series).

Each chapter is complemented with problems, of different complexity. Some solutions and hints are given at the end of the book. The fragments set in smaller type may be omitted on first reading.

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